1991:

1) $f''(x) = 24x - 18$, $f'(1) = -6$, $f(2) = 0$

a) $f'(x) = \int (24x - 18)\,dx = 12x^2 - 18x + C$

$f'(1) = 6$
$-6 = 12 - 18 + C$
$C = 0$

$f'(x) = 12x^2 - 18x$

when horizontal $f'(x) = 0$

$12x^2 - 18x = 0$
$6x(2x - 3) = 0$
$x = 0, \; x = \frac{3}{2}$

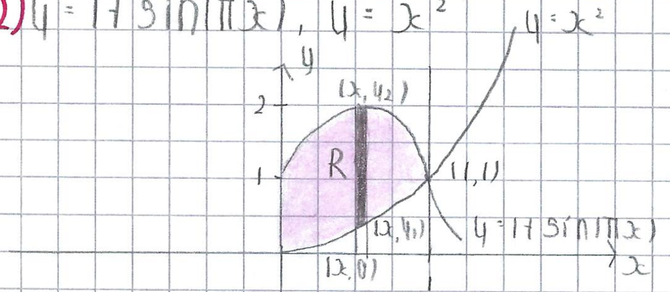
b) $f(x) = \int (12x^2 - 18x)\,dx = 4x^3 - 9x^2 + C$

$f(2) = 0$
$0 = 4(8) - 9(4) + C$
$C = 4$

$f(x) = 4x^3 - 9x^2 + 4$

c) $f(x)_{AV} = \frac{1}{3-1} \int_1^3 (4x^3 - 9x^2 + 4)\,dx$
$= \frac{1}{2}\left[x^4 - 3x^3 + 4x\right]_1^3$
$= \frac{1}{2}\left[\left(3^4 - 3^4 + 4(3)\right) - \left(1 - 3 + 4\right)\right]$
$= \frac{1}{2}(12 - 2)$
$= 5$

2) $y = 1 + \sin(\pi x)$, $y = x^2$



$x^2 = 1 + \sin(\pi x)$

a) Area $R = \int_0^1 (1 + \sin(\pi x) - x^2)\,dx$
$= \left[x - \frac{\cos(\pi x)}{\pi} - \frac{x^3}{3}\right]_0^1$
$= 1 - \frac{\cos\pi}{\pi} - \frac{1}{3} + \frac{\cos 0}{\pi}$
$= 1 + \frac{1}{\pi} - \frac{1}{3} + \frac{1}{\pi} = \frac{2}{3} + \frac{2}{\pi}$

b) $R(x) = y_2 - 0 = 1 + \sin(\pi x)$
$r(x) = y_1 - 0 = x^2$

volume $= \pi \int_0^1 ((1 + \sin(\pi x))^2 - x^4)\,dx$

c) Shell Method - can't do

3) $f(x) = (1 + \tan x)^{3/2}$ for $-\frac{\pi}{4} < x < \frac{\pi}{2}$

a) $f'(x) = \frac{3}{2}(1 + \tan x)^{1/2}(\sec^2 x)$
$= \frac{3}{2}\sec^2 x (1 + \tan x)^{1/2}$

$f(0) = (1 + \tan 0)^{3/2} = 1$

$f'(0) = \frac{3}{2}\sec^2 0 (1 + 0)^{1/2} = \frac{3}{2}(1) = \frac{3}{2}$

Equ. of tangent:
$y - 1 = \frac{3}{2}(x - 0)$
$y = \frac{3}{2}x + 1$
$2y = 3x + 2$
$3x - 2y + 2 = 0$

b) $f(0.02) \approx \frac{3}{2}(0.02) + 1 = 1.03$
The tangent at $(0,1)$ approximates the function at that point

c) $y = (1 + \tan x)^{3/2}$
$x = (1 + \tan y)^{3/2}$
$1 + \tan y = x^{2/3}$
$\tan y = x^{2/3} - 1$
$y = \tan^{-1}(x^{2/3} - 1)$

4) $f(x) = \frac{|x| - 2}{x - 2}$

$f(x) = \begin{cases} \frac{x-2}{x-2}, & x \geq 0 \\ \frac{-x-2}{x-2}, & x < 0 \end{cases}$

a) $\frac{|x| - 2}{x - 2} = 0$
$|x| = 2$, $x - 2 \neq 0$
$x = \pm 2$, $x \neq 2$

$x = -2$

$\lim_{x \to \infty} f(x) = 1$
$\lim_{x \to -\infty} f(x) = -1$

b) $x \geq 0$, $f(x) = \frac{x-2}{x-2} = 1$
$x < 0$, $f(x) = \frac{-x-2}{x-2}$

$f'(1) = 0$

$x < 0, \ f(x) = \dfrac{-x-2}{x-2}$

$f'(x) = \dfrac{(x-2)(-1) - (-x-2)(1)}{(x-2)^2}$

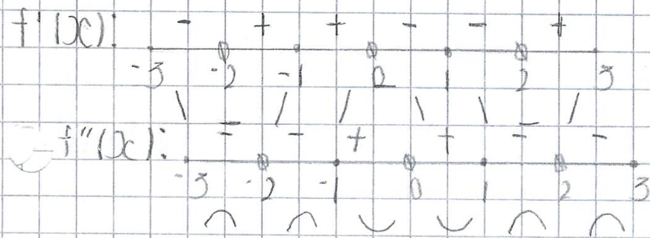
$= \dfrac{-x+2+x+2}{(x-2)^2}$

$= \dfrac{4}{(x-2)^2}$

$f'(-1) = \dfrac{4}{(-3)^2} = \dfrac{4}{9}$

d) Range: $-1 \le y \le 1$

5) $f(0) = 1, \ f(1) = 0, \ f(2) = -1$

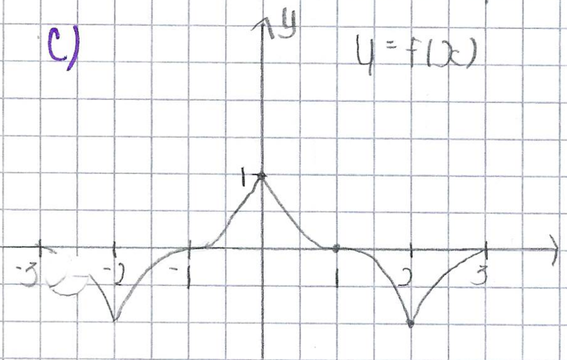


min @ $x = -2, \ x = 2$
max @ $x = 0$

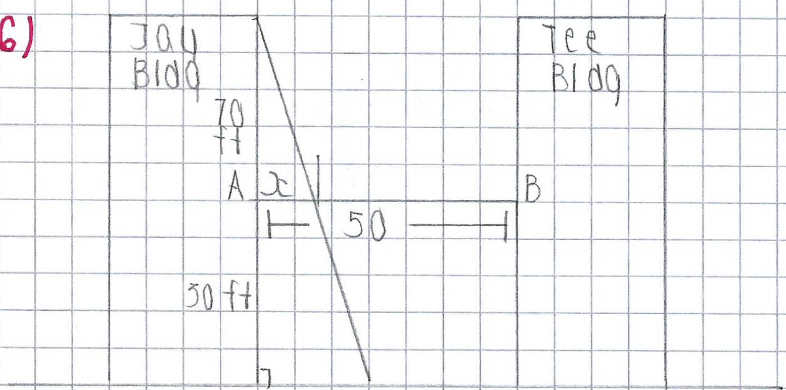
a) Abs. max at
Abs. min at

b) Points of inflection when $f''(x)$ changes concavity
$x = \pm$ (horizontal points of inflection)

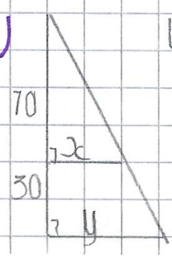
c)



6)



a)

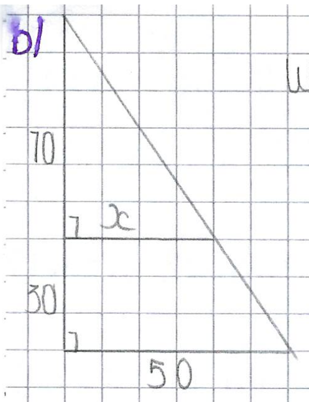


We want $\dfrac{dy}{dt}$, when $x = 25$

$\dfrac{x}{y} = \dfrac{70}{100} \Rightarrow 100x = 70y \Rightarrow 100\dfrac{dx}{dt} = 70\dfrac{dy}{dt} \Rightarrow 100(2) = 70\dfrac{dy}{dt} \Rightarrow \dfrac{dy}{dt} = \dfrac{200}{70} = \dfrac{20}{7}$ ft/

b)

When $y = 50$



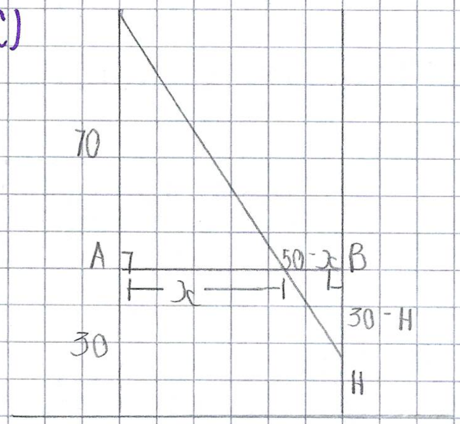
$$\frac{70}{x} = \frac{100}{50}$$

$$\frac{70}{x} = 2$$

$$2x = 70$$

$$x = 35 \text{ ft}$$

c)



$$\frac{70}{x} = \frac{30 - H}{50 - x}$$

$$70(50 - x) = x(30 - H)$$

$$30 - H = \frac{3500 - 70x}{x}$$

$$H = 70 - \frac{3500}{x} + 30 \qquad x \neq 0$$

$$H = 100 - 3500x^{-1}$$

$$\frac{dH}{dx} = \frac{3500}{x^2}$$

When $x = 40$:

$$\frac{dH}{dt} = \frac{dH}{dx} \cdot \frac{dx}{dt} = \frac{3500}{x^2} \cdot 2 = \frac{3500}{(40)^2} \cdot 2 = \frac{3500}{800} = \frac{35}{8} = 4\tfrac{3}{8} \text{ ft/sec}$$